Unemployment and Migration in a Currency Union[1]

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Abstract

How does migration within a currency union affect welfare across the union? I study this question in this paper with a New Keynesian currency union model. The union consists of two countries whose economies are characterized by labor market frictions. One country member has a higher job-finding rate and a lower unemployment rate compared to the other country, hence unemployed agents in the latter have an incentive to relocate to the former. I show that when firms have the ability to hire workers from abroad and when unemployed agents can relocate to a different country, the negative impact of asymmetric shocks is significantly reduced, improving welfare across the union on average.

Keywords: Labor Mobility, Unemployment, Currency Union
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1 Introduction

This paper attempts to fill the gap in the literature between labor market frictions and labor mobility in a currency union. The model I develop is a simple currency model with labor market friction and a constant rate of migration from one country member to another country member of the currency union. The research question is whether or not labor migration within a monetary union improves economic conditions across the union. The theoretical literature about labor mobility within a currency union is very sparse, although it has been discussed as a precondition for the optimality of the union since Mundell [1961]. The idea is very straightforward: if one region in the union is in a recession and another region is enjoying better economic conditions, then unemployed workers from the region in recession can relocate to the booming region, improving welfare for the entire union. High labor mobility within the US, a currency union, is cited as the reason for improvement of economic conditions during some past periods of regional recession. For instance Table 1 below reports some labor statistics (unemployment rates and share of employment) for the state of Massachusetts during and after a period of regional economic slowdown in the late eighties. While the rest of the US enjoyed relatively higher economic activity, Massachusetts was going through difficult economic times (with unemployment rate peaking at around 8.8%). By 1996 the region had recovered and achieved full employment, but the number of employed, as measured by the state's share in US total employment, never reached the pre-bust levels. Even though unemployment was down to the same level as 1986, the employment share in 1996 was lower than the 1986 figure. This indicates that the unemployed agents simply relocated to different parts of the country for better job prospects. Low levels of labor mobility within the euro area [Bonin et al., 2008] are seen by many scholars as the reason for the recent episodes of extended stagnation experienced by some countries in the union.

Table 1: Labor Mobility and Unemployment Krugman [2012]

Year	MA Share in US Employment	MA Unemployment	US Unemployment
1986	2.70	4.0	7.0
1991	2.48	8.8	6.8
1996	2.43	4.6	5.4

Farhi and Werning [2014] and Dmitriev and Hoddenbagh [2015] were arguably the first to formally study this topic in a theoretical model. Farhi and Werning [2014] show that labor mobility does not necessarily improve the welfare of the union after agents relocate to different parts. In their model, agents who relocate achieve higher economic welfare, but their relocation does not necessarily improve welfare for those who stayed behind. In their view, labor mobility does not necessarily improve economic welfare for the union. Dmitriev and Hoddenbagh [2015] allow for labor mobility in a dynamic model and conclude that it eliminates some distortions in the economy, namely real wage rigidity and the lack of risk sharing (if financial autarky).

In this paper, I set up a New Keynesian (NK), dynamic stochastic general equilibrium (DSGE) that extends the benchmark model developed by Abbritti and Mueller [2013].

The model consists of a simple currency union with country members Home and Foreign. Both country members' economies are characterized by labor market frictions as developed by Mortensen and Pissarides [1994] and introduced in a NK model by Blanchard and Galí [2010]. The extension I introduce is a constant migration rate from Foreign (the country member with the higher long-run unemployment rate and fewer job prospects) to Home (the country with the lower long-run unemployment rate and more job prospects for unemployed agents). One can think of Greece or Spain being the Foreign country and Germany or the United Kingdom being the Home country. In fact, I calibrate the model to match the labor market moments of Germany and Greece. I only consider one-way migration (from Foreign to Home) for simplicity and tractability. Furthermore, I take monetary policy as given and study the impact of migration on short run dynamics and the transmission of aggregate shocks across country members of the monetary union.

I find that labor migration increases the job-finding rate in the country of origin (Foreign) and decreases the job-finding rate in the host country (Home). This causes Home unemployment to rise and Foreign unemployment to fall. The relocating agents increase the size of the pool of job seekers in Home and decrease the size of the pool of job seekers in Foreign. With a constant numbers of job openings, the rise of the number of job seekers increases unemployment in Home and the fall in the number of job seekers lowers unemployment in Foreign. I also find that migration of unemployed agents increases welfare for the host country and decreases welfare for the country of origin. On average, the union's welfare improves because the welfare gains in Home outweigh the losses in Foreign. In Home, higher unemployment means lower welfare, but a lower job-finding rate means lower hiring costs. Lower hiring costs mean that firms increase employed labor and thus output expands and consumption also expands. The positive impact of the increase in consumption is larger than the negative impact of the increase in labor effort (employment and unemployment). The opposite is true in Foreign. It is important to note that neither nominal wages nor real wages are equalized across country members. This is due to the fact that there is no perfect labor mobility, but rather an exogenous constant rate of migration, hence the features and distortions creating diverging wages are still present.

This paper contributes to the literature on labor mobility within currency unions, with the main contribution being the inclusion of unemployment as a motive for migration. Past studies of labor mobility have ignored unemployment Dmitriev and Hoddenbagh [2015], (Baglioni et al. [2015] and Farhi and Werning [2014]). And the few studies that have considered unemployment, have ignored migration of unemployed agents between country members of the union. This paper attempts to close this gap by studying unemployment in a currency union with an exogenous migration rate. A significant amount of the debate regarding the current euro crisis is centered around the level of unemployment in different parts of the union (i.e with high unemployment in Spain and low unemployment in Germany), and no model of currency union formally incorporates unemployment.

Section 2 briefly reviews the literature on labor market friction and labor mobility in a currency union. Section 3 lays out the model. Sections 4 and 5 present the constrained efficient allocation and the Nash bargained wage. Section 9 explains the calibration and Section 10 compares the short dynamics of the union with labor migration and the economy with no labor migration. Section 12 concludes.

2 In the Literature

The optimal currency area (OCA) literature pioneered by Mundell [1961] extensively documents factor (i.e capital and labor) mobility as a precondition for an optimal currency area. Though the idea has been around for a long time, relatively few formal models have been developed to explain how labor mobility helps with macroeconomic adjustment.

Farhi and Werning [2014] develop a very stylized static model of a currency union and conclude that labor mobility does not necessarily improve welfare cross the union. On one hand, if the region experiencing an economic downturn has specialized production and consumption (non-traded), then relocating agents achieve higher welfare, but those who stay do not. On the other hand, if the consumption is not specialized in the region experiencing a downturn, then relocating agents improve the welfare of those who stay. Contrary to Farhi and Werning [2014], Dmitriev and Hoddenbagh [2015] develop a dynamic model (both of these studies do not include unemployment), and conclude that labor mobility can act as a substitute to fiscal integration within the currency union.

Baglioni et al. [2015] study labor mobility in a currency union within a stylized model and conclude that labor mobility is not a precondition for the union to be optimal. Particularly, they show that costless labor mobility does not necessarily improve welfare. Mandelman and Zlate [2012] study migration of unskilled labor from Mexico to the US and the patterns of remittances sent back to Mexico by those migrants. The model documents and explains short run dynamics and aggregate shock transmission in the presence of migration of unskilled labor. The closest model to the one I develop in this paper is that of Abbritti and Mueller [2013], where the authors develop a simple model of a currency union with labor market frictions, which gives rise to involuntary unemployment. The model in this paper can be thought of as a merger of the models in Mandelman and Zlate [2012] and Abbritti and Mueller [2013].

Although I do not study optimal policy in this paper, it is worth mentioning the vast literature on optimal fiscal and monetary policy in a currency union, which has extensively been studied in recent years. Benigno [2004] characterizes the optimal monetary policy in a currency union in which country members are characterized by different degrees of nominal rigidity. Ferrero [2009] and Galí and Monacelli [2008] lay out the optimal conduct of both fiscal and monetary policy in a currency union. All concluded that price stability at the union level is optimal and fiscal policy should be used for country specific counter-cyclical stabilization. The research papers in this area of the literature have not incorporated unemployment or labor mobility in their studies. Blanchard and Galí [2010] explain the importance of unemployment in discussing monetary policy, which has been the main criticism and shortcoming of traditional New-Keynesian models. Along the lines of Blanchard and Galí [2010], Faia [2009] study the optimal Ramsey monetary policy with labor market frictions and conclude that there is a departure from price stability in the presence of labor market frictions. In what follows I lay out the elements of the model I develop in order to study the implications of labor migration.

3 The Model

3.1 Households

The household in country j maximizes its lifetime utility:

$$E_0 \sum \beta^t \left(logC_t^j - \frac{\chi}{1+\phi} \left(L_t^j\right)^{1+\phi} \right) \tag{1}$$

where the consumption basket in each country is defined as $C_t = \left(\frac{C_{H,t}}{1-\alpha}\right)^{1-\alpha} \left(\frac{C_{F,t}}{\alpha}\right)^{\alpha}$ and $C_t^* = \left(\frac{C_{H,t}^*}{1-\alpha}\right)^{1-\alpha} \left(\frac{C_{F,t}^*}{\alpha}\right)^{\alpha}$. $L_t = N_t + \kappa U_t$ is the total labor effort by members of the representative household. This captures the fact that the household derives a dis-utility from employed and unemployed agents. N_t is employment and U_t is unemployment. A star (*) denotes variables for the Foreign country and H, F denotes the country of origin (i.e $C_{H,t}$ is the consumption bundle produced in Home and consumed in Home; $C_{H,t}^*$ is the consumption bundle produced in Home and consumed in Foreign).
The budget constraint for the Foreign country is the following equation:

$$P_t^* C_t^* + E_t \left(Q_{t,t+1}^* D_{t+1}^*\right) + \delta^*(1-\upsilon) N_t^* f_{e,t} = W_t^* N_t^* + D_t^* \tag{2}$$

where W_t^* is the Nash equilibrium bargained wage, which I'll return to later in the paper. $\delta^*(1-\upsilon)$ represents the fraction of agents who lost their jobs and decided to relocate to Home. $\delta^*(1-\upsilon)N_t^* f_{e,t}$ is the cost associated with relocating unemployed household members, which is expressed in terms of bundles of Foreign goods. And $f_{e,t} = \varepsilon_t f_e$ where ε_t is the shock to the fixed migration cost (i.e random subsidies that lower the cost of moving or restrictions that increase the cost of migration). This is modeled after Mandelman and Zlate [2012]. The total cost of relocation (paid by the Foreign household) is $F_t^* = \delta^*(1-\upsilon)N_t^* f_{e,t}^*$ and is expressed in terms of units of Foreign (produced) goods. Finally for simplicity, I assume that financial markets are complete. The (Foreign) households' optimality conditions are:

$$\chi C_t^* \left(L_t^*\right)^{\phi} = \frac{W_t^* - \delta^*(1-\upsilon) f_{e,t}}{P_t^*} \tag{3}$$

$$\beta \left(\frac{C_t^*}{C_{t+1}^*}\right) \left(\frac{P_t^*}{P_{t+1}^*}\right) = \frac{1}{R_t} = Q_{t,t+1} \tag{4}$$

The optimality conditions for the representative household in Home is standard and exactly the same as the above condition when $\upsilon = 1$, meaning no migration.

3.2 Relative Prices and International Risk Sharing

The terms of trade between the Home and Foreign countries are given by:

$$S_t = \frac{P_{F,t}}{P_{H,t}} \tag{5}$$

The Law of One Price holds across the union, so $P_{F,t} = P_{F,t}^*$ and $P_{H,t} = P_{H,t}^*$. The aggregate prices in the Home and Foreign countries are respectively $P_t = P_{H,t}^{1-\alpha} P_{F,t}^{\alpha}$ and $P_t^* = \left(P_{H,t}^*\right)^{\alpha} \left(P_{F,t}^*\right)^{1-\alpha}$. These can be rewritten as a function of the terms of trade as $P_t =$

$P_{H,t}S_t^{\alpha}$ and $P_t^* = P_{F,t}S_t^{-\alpha}$. The real exchange rate is then given by $Q_t = P_t^*/P_t = S_t^{1-2\alpha}$. In complete asset markets, each household has access to a complete set of contingent claims, traded internationally. It follows that Home and Foreign consumption co-move with respect to the real exchange rate:

$$C_t = \Upsilon Q_t C_t^* \tag{6}$$

3.3 Firms and the Labor Market

3.3.1 The Labor Market

At the beginning of every period, a fraction N_t of any given household's members are employed and the remaining members $1-N_t$ are unemployed. Of those who are employed, a fraction δ lose their job by the end of the period. In the Foreign country a fraction $(1-\upsilon)$ of those who lose their jobs relocate to the Home country to find employment. In Home, the pool of jobless members then consists of those unemployed plus those who arrive from Foreign.

Timing and Flow:

The pool of employed in country i consists of newly hired agents and those who remained employed last period. This pool evolves as:

$$N_{i,t} = (1-\delta_i)\, N_{i,t-1} + H_{i,t} \tag{7}$$

The size of the pool of newly hired agents (the difference between employment at time t and employment at the end of time t-1) is also given by the above equation: $H_{i,t} = N_{i,t} - (1-\delta_i)\, N_{i,t-1}$. The pool of jobless agents looking for employment at the end of each period is given by:

$$U_{i,t} = (Lf_{i,t} - N_{i,t-1}) + \delta_i N_{i,t-1} = Lf_{i,t} - (1-\delta_i)\, N_{i,t-1} \tag{8}$$

However, the aggregate pool of jobless looking for work in each country is different than the above equation due to migration. A fraction of this pool from Foreign relocates to Home and the pool of unemployed agents looking for employed in Home is given by:

$$U_t^0 = [Lf_{ss} - (1-\delta)\, N_{t-1}] + (1-\upsilon)\left[Lf_{ss}^* - (1-\delta^*)\, N_{t-1}^*\right] \tag{9}$$

Home labor market also depends on Foreign labor market conditions. The size of the total pool of jobless looking for work in Foreign (from Home and Foreign) is:

$$U_t^{*0} = \upsilon\left[Lf_{ss}^* - (1-\delta^*)\, N_{t-1}^*\right] \tag{10}$$

With migration, the labor force in both countries changes. As the labor increases in Home with migration, it falls in Foreign.

$$Lf_t = Lf_{ss} + (1-\upsilon)\left[Lf_t^* - (1-\delta^*)\, N_{t-1}^*\right] \tag{11}$$

$$Lf_t^* = \frac{Lf_{ss}^* + (1-\upsilon)(1-\delta^*)\, N_{t-1}^*}{(2-\upsilon)} \tag{12}$$

The labor force in foreign decreases by the number of unemployed who relocate.

Hiring Costs:

Every firm in both countries must pay to hire new workers; the cost of hiring one worker is:

$$G_{i,t} = \Gamma_i A_{i,t} x_{i,t}^{\gamma} \tag{13}$$

Total hiring costs for firm m in country i is $G_{i,t}H_{i,t}(m)$. There is no uncertainty in hiring costs and the costs are expressed in terms of domestic goods. The (per hire) cost can be expressed as $G_{i,t} = \Gamma_i A_{i,t}\left(\frac{H_{i,t}}{E_{i,t}}\right)^{\gamma}$. Furthermore, $x_{i,t} = \left(\frac{H_{i,t}}{U_{i,t}^{0}}\right)$ is a measure of labor market tightness in country i, also referred to as the job finding rate. For each country the job-finding rate can expressed as:

$$x_t = \frac{N_t - (1-\delta)N_{t-1}}{\left[Lf_t - (1-\delta)N_{t-1}\right] + (1-\upsilon)\left[Lf_t^* - (1-\delta^*)N_{t-1}^*\right]} \tag{14}$$

$$x_t^* = \frac{N_t^* - (1-\delta)N_{t-1}^*}{\upsilon\left[Lf_t - (1-\delta^*)N_{t-1}^*\right]} \tag{15}$$

For Home, the hiring cost is now a function of both Home and Foreign labor market characteristics.

3.4 Market Clearing Conditions

Focusing on goods market clearing for Foreign, output must equal the sum of Foreign demand for Foreign goods, Home demand for Foreign goods, the total hiring cost, the cost of relocation to Home and the cost of price adjustment (all expressed in terms of bundles of Foreign goods). Firm m in Foreign faces the following aggregate demand for its good:

$$Y_t(m)^* = \left(\frac{P_{F,t}(m)}{P_{F,t}}\right)^{-1}\left(C_{F,t} + C_{F,t}^* + G_t^* H_t^* + F_t^* + \Psi_t^*\right) \tag{16}$$

Using the definition of aggregate output, $Y_t = \left(\int_0^1 (Y_t(m))^{\frac{\epsilon-1}{\epsilon}} dm\right)^{\frac{\epsilon}{\epsilon-1}}$, the market clearing condition becomes:

$$Y_t^* = C_t^* S_t^{-\alpha} + G_t^* H_t^* + F_t^* + \Psi_t^* \tag{17}$$

Expressed in terms of Home consumption $Y_t^* = C_t S_t^{\alpha-1} + G_t^* H_t^* + F_t^* + \Psi_t^*$. A similar market clearing condition for the Home country can be derived:

$$Y_t = C_t S_t^{\alpha} + G_t H_t + \Psi_t$$

4 The Constrained Efficient Allocations

I first look at the equilibrium in the absence of all nominal rigidity and monopolistic power. The social planner takes the labor market frictions as given and allocates resources accordingly, by maximizing the union-wide welfare with respect to the resource constraints in both Home and Foreign. The union-wide welfare is simply the sum of Home and Foreign welfare functions:

$$U_t + U_t^* = \sum_{t=0}^{\infty} \beta^t \left(logC_t - \frac{\chi}{1+\phi}(L_t)^{1+\phi} + logC_t^* - \frac{\chi^*}{1+\phi}(L_t^*)^{1+\phi}\right) \tag{18}$$

The resource constraints are as defined earlier. The constrained efficient allocation of consumption is equalized consumption across Home and Foreign: $C_t = C_t^*$. Given a job finding rate x, the constrained efficient level of employment is:

$$N_e = \frac{x\left(1+(1-\upsilon)\left[1-(1-\delta^*)\,N_e^*\right]\right)}{\delta+(1-\delta)\,x} \tag{19}$$

$$N_e^* = \frac{\upsilon x^*}{\delta+(1-\delta^*)\,\upsilon x^*} \tag{20}$$

The constrained optimal job finding rate is given by the solution of the following system of equations:

$$\begin{aligned}\chi C_t N_t^{\phi} = &[1-\Gamma(\gamma+1)x_t^{\gamma}] + ...\\ &\Gamma\left(\frac{A_{t+1}}{A_t}\right)\left(\frac{C_t}{C_{t+1}}\right)\left[-(\gamma+1)(1-\delta)x_{t+1}^{\gamma}+\gamma\upsilon(1-\delta)x_{t+1}^{\gamma+1}\right] + ...\\ &\Gamma^*\left(\frac{A_{t+1}^*}{A_t}\right)\left(\frac{C_t}{C_{t+1}^*}\right)\gamma\,(1-\upsilon)\,(1-\delta)\,x_{t+1}^{*(\gamma+1)}\end{aligned}$$

$$\begin{aligned}\chi^* C_t^* N_t^{*\phi} = &[1-\Gamma^*(\gamma+1)x_t^{*\gamma}] + ...\\ &\Gamma^*\left(\frac{A_{t+1}^*}{A_t^*}\right)\left(\frac{C_t^*}{C_{t+1}^*}\right)\left[-(\gamma+1)(1-\delta^*)x_{t+1}^{*\gamma}+\gamma\upsilon^*(1-\delta^*)x_{t+1}^{*\gamma+1}\right]\end{aligned}$$

where both N_t and N_t^* are functions of job finding rates x_t and x_t^*. The constrained optimal rate of migration υ is then the following equation:

$$\upsilon = \frac{\left[1+\left(\frac{(1-(1-\delta)N_{t-1})}{\left(1-(1-\delta^*)N_{t-1}^*\right)}\right)\right]}{\left[1+\left(\frac{C_t*}{C_t}\right)\left(\frac{\Gamma^*}{\Gamma}\right)\left(\frac{A_t}{A_t^*}\right)\left(\frac{\gamma}{\gamma^*}\right)\left(\frac{x_t^{\gamma}}{(x_t^*)^{\gamma^*}}\right)\right]} \tag{21}$$

5 Wage Bargaining

Here I focus on wage bargaining in Foreign and relegate wage bargaining in Home to the appendix at the end of the paper. The value of an employed household member in the Foreign country V_t^{*E} is:

$$V_t^{*E} = \widetilde{W_t^*} - \chi^* C_t^* L_t^{*\phi} + \beta E_t\left\{\frac{C_t^*}{C_{t+1}^*}\left[\left(1-\delta^*\upsilon(1-x_{t+1}^*)\right)V_{t+1}^{*E}+\delta^*\upsilon(1-x_{t+1}^*)V_{t+1}^{*U}\right]\right\} \tag{22}$$

The value of the employed is now a function of υ, and when there is no migration ($\upsilon = 1$) the equation reduces to the ones found in the literature on labor market frictions ([Blanchard and Galí, 2010]). The value of an unemployed household member in Foreign V_t^{*U} is also a function of migration in addition to the cost of migration to Home:

$$V_t^{*U} = -\delta^*(1-\upsilon)f_{e,t}^* + \beta E_t\left\{\frac{C_t^*}{C_{t+1}^*}\left[x_{t+1}^* V_{t+1}^{*E}+(1-x_{t+1}^*)V_{t+1}^{*U}\right]\right\} \tag{23}$$

Combining the employed and unemployed values, the households' surplus can expressed as:

$$\begin{aligned}V_t^{*E} - V_t^{*U} = &\widetilde{W_t^*} - \chi^* C_t^* L_t^{*\phi} + \delta^*(1-\upsilon)f_{e,t}^* + ...\\ &(1-\delta^*\upsilon)\,\beta E_t\left\{\frac{C_t^*}{C_{t+1}^*}\left[(1-x_{t+1}^*)\left(V_{t+1}^{*E}-V_{t+1}^{*U}\right)\right]\right\}\end{aligned} \tag{24}$$

The value of an employee to a firm (continued match) V_t^{*F} is just the hiring cost in terms of bundles of the Foreign good:

$$V_t^{*F} = \left(\frac{P_{F,t}^*}{P_t^*}\right) G_t^* = A_t^* \Gamma^* S_t^\alpha x_t^{*\gamma} \tag{25}$$

This surplus will be split between the firm and the workers through bargaining. Furthermore, if μ is the relative bargaining power of the workers then:

$$V_t^{*E} - V_t^{*U} = \mu V_t^{*F} = \mu^* A_t^* \Gamma^* S_t^\alpha x_t^{*\gamma} \tag{26}$$

The Nash wage schedule in the Foreign country is given by the following three conditions:

$$\begin{aligned} \widetilde{W_t^*} = \mu^* A_t^* \Gamma^* S_t^\alpha x_t^{*\gamma} + \chi^* C_t^* L_t^{*\phi} - \delta^*(1-\upsilon) f_{e,t}^* - ... \\ (1-\delta^*\upsilon)\,\beta E_t \left\{ \frac{C_t^*}{C_{t+1}^*} \left[(1-x_{t+1}^*) \left(\mu^* A_{t+1}^* \Gamma^* S_{t+1}^\alpha x_{t+1}^{*\gamma} \right) \right] \right\} \end{aligned} \tag{27}$$

6 Price Setting

6.1 Flexible Prices

Under flexible prices, the firms set their prices as a mark-up $\left(\varrho = \frac{\epsilon}{\epsilon-1}\right)$ over their marginal cost.

$$P_{H,t}(i) = \varrho MC_t P_{H,t} \tag{28}$$

where MC_t is the real marginal cost in terms of bundles of the domestic goods. The (supply-side) flexible price equilibrium condition in Home is given by:

$$MC_t^r = \frac{\widetilde{W_t}}{A_t} S_t^\alpha + \Gamma x_t^\gamma - \beta(1-\delta) E_t \left\{ \left(\frac{C_t}{C_{t+1}}\right) \left(\frac{S_t}{S_{t+1}}\right)^\alpha \left(\frac{A_{t+1}}{A_t}\right) \Gamma x_{t+1}^\gamma \right\} \tag{29}$$

Note that in equilibrium, assuming symmetry, $P_{H,t}(j) = P_{H,t}$ and $MC_t = \frac{1}{\varrho}$. Replacing marginal cost in equation 29 with the previous expression, the supply-side equilibrium is given by the following condition:

$$\frac{\widetilde{W_t}}{A_t} S_t^\alpha = \frac{1}{\varrho} - \Gamma x_t^\gamma + \beta(1-\delta) E_t \left\{ \left(\frac{C_t}{C_{t+1}}\right) \left(\frac{S_t}{S_{t+1}}\right)^\alpha \left(\frac{A_{t+1}}{A_t}\right) \Gamma x_{t+1}^\gamma \right\} \tag{30}$$

Under flexible prices, monetary policy is neutral, hence I introduce nominal rigidity (sticky prices) so that monetary policy plays a stabilizing role.

6.2 Sticky Prices

I introduce price stickiness à la Rottemberg. Every period, each firm faces a quadratic cost of price adjustment. Each firm m in country j chooses $P_{j,t}(m)$ to maximize its lifetime profits. Firm m in the Home chooses the optimal price by solving the following maximization problem:

$$\begin{aligned} \underset{P_{H,t}(m)}{Max}\, E_t \sum_0^\infty Q_{t,t+k} \left\{ P_{H,t+k}(m) Y_{H,t+k}(m) - MC_{H,t+k}^r Y_{H,t+k}(m) P_{H,t+k} - ... \right\} \\ \left\{ ... \frac{\psi}{2} \left(\frac{P_{H,t+k}(m)}{P_{H,t+k-1}(m)} - 1 \right)^2 Y_{H,t+k} P_{H,t+k} \right\} \end{aligned}$$

using the fact that all firms get to reset their prices and thus choose the same optimal price, which must satisfy the following equation:

$$(1-\epsilon)+\epsilon MC^{r}_{H,t}-\psi\left(\frac{P_{H,t}}{P_{H,t-1}}-1\right)\left(\frac{P_{H,t}}{P_{H,t-1}}\right)+...$$
$$...\psi Q_{t,t+1}\left(\frac{P_{H,t+1}}{P_{H,t}}-1\right)\left(\frac{Y_{H,t+1}}{Y_{H,t}}\right)\left(\frac{P_{H,t+1}}{P_{H,t}}\right)^{2}=0$$

The previous equation can then be expressed in terms of inflation to obtain the (supply-side) sticky price equilibrium, and is represented by the following equation (the Philips curve):

$$(1-\epsilon)+\epsilon MC^{r}_{H,t}-\psi\left(\Pi_{H,t}-1\right)\Pi_{H,t}+\psi Q_{t,t+1}\left(\Pi_{H,t+1}-1\right)\left(\frac{Y_{H,t+1}}{Y_{H,t}}\right)\Pi^{2}_{H,t+1}=0 \quad (31)$$

These conditions are similar for both countries. The following equations describe the situation in the Foreign country:

$$MC^{*r}_{t}=\frac{\widetilde{W^{*}_{t}}}{A^{*}_{t}}S^{-\alpha}_{t}+\Gamma^{*}x^{*\gamma}_{t}-\beta\left(1-\delta^{*}\right)E_{t}\left\{\left(\frac{C^{*}_{t}}{C^{*}_{t+1}}\right)\left(\frac{S_{t}}{S_{t+1}}\right)^{-\alpha}\left(\frac{A^{*}_{t+1}}{A^{*}_{t}}\right)\Gamma^{*}x^{*\gamma}_{t+1}\right\} \quad (32)$$

$\widetilde{W^{*}_{t}}$ is the Nash bargained wage given by equation 27.

7 Real Wage Rigidity

Without real wage rigidity, unemployment remains constant. To induce inefficient fluctuations in the unemployment, I follow the literature and introduce real wage rigidity. Different approaches have been used to model real wage rigidity and all of them have their drawbacks. Here I follow Hall [2005] and Blanchard and Galí [2010] and assume that the real wage follows a wage norm:

$$W^{r}_{t}=\left(\widetilde{W_{t}}\right)^{1-\nu}\left(W_{ss}\right)^{\nu}$$
$$W^{*r}_{t}=\left(\widetilde{W_{t}}^{*}\right)^{1-\nu^{*}}\left(W^{*}_{ss}\right)^{\nu^{*}}$$

The real wages are a Cobb-Douglas aggregation of the Nash bargained wage and the wage norm. The wage norm in each country is simply the steady state level of the Nash bargained wages, hence in the steady state, the real wage is the same as the Nash bargained wage.

8 Monetary Policy

The equilibrium conditions of this simple currency union model are described in the Appendix. To close the model a monetary policy rule must be specified. I do not characterize the optimal monetary policy, but instead I follow Faia [2009] and Baglioni et al. [2015]3 and set the monetary policy rule as:

$$R_{t}=\left(\frac{1}{\beta}\right)\left(\frac{Y_{t}}{Y_{ss}}\right)^{\phi_{y}}\left(\frac{\Pi_{t}}{\Pi_{ss}}\right)^{\phi_{\pi}} \quad (33)$$

All of the variables are averages of those of the country members, and the central bank conducts monetary policy for the whole union. Y_{ss} is the union-wide average steady level of output and Π_{ss} is the union-wide average steady level of inflation, which I assume to be equal to one.

9 Calibration

Each time period corresponds to a quarter. I set the parameters that characterize the preferences equal to the common values used in the literature.

$$\beta = 0.96 \qquad \phi = 5$$

This corresponds to a long-run interest rate of about 4% and a Frisch elasticity of 0.2. I assume these parameters to be uniform across both countries in the union.

Many other estimates have been reported in the literature, but alternative estimates have no impact on the results of this paper since the focus is not monetary policy but instead labor mobility.

Below I describe the calibration strategy used to pin down the parameters describing the labor markets in each country. Table 3 in the Appendix summarizes all of the parameters used in the simulation exercise.

9.1 Euro Area: Germany and Greece

I calibrate the model so that the steady state matches the labor characteristics of two countries in the euro area. One country has a lower steady state unemployment rate and a higher job finding rate. I calibrate the Home country to match Germany and Foreign to match Greece. Germany has the lowest unemployment rate in the euro area and Greece, the highest. The calibration is such that when there is no labor mobility ($\upsilon = 1$), the long-run unemployment rates match those of Germany (Home) and Greece (Foreign). The characteristics of the labor market in both countries are described below.

Germany: the unemployment rate in Germany is $U = 5\%$ and the employment rate stands at 73.91% [3]. This implies a labor force of 78.91%. I take the job finding rate from Hobijn and Sahin [2007] and set $x_{ss} = 20\%$ (a monthly job finding rate of 7%). Given the unemployment and the job finding rates, the separation rate is $\delta = 0.013$. I follow Blanchard and Galí [2010] and assume that the total hiring cost is about 1% of GDP, and the value of Γ is reported in Table 3.

Greece: the unemployment rate in Greece is about $U^* = 26.05\%$ and the employment rate is 49.75%, which implies a labor force participation rate of 75.80%. Hobijn and Sahin [2007] reports a job-finding rate of $x^*_{ss} = 0.151$ for Greece. This is consistent with a monthly job-finding rate of 5.3%. The separation rate imputed from these is $\delta^* = 0.056$. I also assume that the total hiring cost is about 1% of GDP.

The monthly job-finding and separation rates taken from Hobijn and Sahin [2007] are then translated to cumulative quarterly job-finding rates as:

$$x_q = x_m + (1 - x_m)x_m + (1 - x_m)^2 x_m$$

[3] From FRED (St Louis FED)

where x_m is the monthly job-finding rate. And given the job-finding rate and the employment rate, the separation rate for both countries is calculated as:

$$\delta = \frac{x\left[(Lf - N) + (1-\upsilon)\left(Lf^* - (1-\delta^*)N^*\right)\right]}{(1-x)\,N} \tag{34}$$

$$\delta^* = \frac{x^*\upsilon\left(Lf^* - N^*\right)}{(1-x^*\upsilon)\,N^*} \tag{35}$$

The migration rate parameter is free to vary, and I chose two different values in the simulations. The first is such that the job-finding rates are equalized across both countries and unemployed from Greece no longer have an incentive to move in order to increase their job prospects. Under this scenario $\upsilon = 0.97$, so only 3% of the unemployed members relocate every period. The second calibration is such that there is convergence in unemployment rates across the countries, hence long run unemployment rates are equalized. In this case $\upsilon = 0.79$, which means that every period about 30% of the unemployed agents in Greece relocate to Germany. A higher rate of migration is required to achieve convergence in unemployment compared to convergence in the job-finding rate.

ECB's Monetary Policy: As for the parameters in the monetary policy rule, I simply use the specification in Fourcans and Vranceanu [2002] and set $\phi_\pi = 1.16$ and $\phi_y = 0.18$.

10 Short Term Dynamics

In this section, I compare the short-run dynamic adjustments of the economies when Foreign experiences a negative productivity shock under the two different rates of migration mentioned above. Figure 12 displays the impulse responses of some select variables to the negative shock in Foreign (Greece). With a negative productivity shock, output shrinks and unemployment increases in Foreign. Home unemployment also rises due to a portion of the unemployed migrating from Home. The union average unemployment also rises. Except for Home, unemployment rises less with migration, especially when the migration rate is such that long run unemployment is equalized across the two countries.

The negative shock causes a decrease in the job-finding rate in both countries. Again the fall is lessened by migration. Inflation behaves in the same manner regardless of the presence of migration. This result is surprising since the Nash bargained wage is a function of the cost of relocation, so the marginal cost faced by the firm would also be affected by this cost. The migration rate and migration cost both affect the marginal cost, so if their effects cancel each other then inflation would react the same regardless of migration.

Given the dampening effect that migration has on these variables at the union level, one should expect an improvement in the union's average welfare due to migration.

11 Migration and Welfare

In this section I explore the impact of migration on welfare in both countries. I compare welfare by calculating the compensation required in terms of consumption such that the representative household is indifferent between living in an environment with labor mobility and one with no labor mobility. The consumption equivalence percentage Δ

solves the following equation:

$$W_{migration} = \frac{U\left[\left(1+\frac{\Delta}{100}\right)C_{nomigration}, L_{nomigration}\right]}{(1-\beta)} \tag{36}$$

where W is the welfare with labor mobility and the function U on the right hand side is the period utility when there is no labor mobility. I calculate Δ for both country members. When the migration rate is calibrated so that the job-finding rates are equalized in the long run, the welfare loss in Home (Greece) due to labor migration is $\Delta = -0.11\%$. This loss decreases to $\Delta = -0.06\%$ when the migration is such that long run unemployment rates are equalized. There are welfare gains for Foreign (Germany), the host country. The welfare gains are $\Delta = 3.18\%$ when υ is such that $x = x^*$ and $\Delta = 0.67\%$ when the migration rate is such that long run unemployment rate are equalized. The welfare gains and losses associated with migration are summarized in Table 2

Table 2: Welfare Gain/Loss

		Home (Germany)	Foreign (Greece)
$\upsilon = 0.793$	(s.t $Un = Un^*$)	3.18%	-0.11%
$\upsilon = 0.97$	(s.t $x = x^*$)	0.67%	-0.06%

On average, the union's welfare increases in both calibrations.

12 Conclusion

This paper revisited the topic of labor mobility in a currency union and in particular attempted to answer the question of whether labor mobility helps the union's economy adjust more quickly to aggregate shocks in the short run, thus improving welfare across the union. While some existing studies have concluded that labor mobility improves welfare, others have concluded the opposite. These past studies have all ignored unemployment and the role of labor market characteristics in determining labor migration and its impact on the economy. In this paper, I set up a simple model of a currency union with two country members characterized by different labor markets frictions. I calibrate the model to match the labor market moments of the economies of Germany and Greece. Every period, a constant fraction of unemployed agents in Greece relocate to Germany. I first calibrate the rate of migration such that the job-finding rate is equalized across country members in the long run. In an alternative calibration, I pick the rate of labor migration so that the job-finding rates are equalized in the long run. Under both calibrations I find that labor mobility decreases unemployment in the country of origin but increases the job finding rate, and welfare falls. The exact opposite happens in the host country: unemployment rises the job finding rate falls, and welfare improves. Assuming that households are identical across country members, then on balance, labor migration is welfare improving for the union as a whole since the welfare gains in the host country outweigh the losses in the country of origin.

This paper provides several opportunities for several extensions. First, the natural continuation of this work would be the study of the optimal fiscal and monetary policy, which would provide insight into how limited migration, as it is this paper, influences the conduct of the optimal policy. A second extension, which I plan to tackle in future research, is the possibility of different consumption baskets for migrants in the host country. This extension could prove to be welfare-improving for both the country of origin of the migrants and also for the host country. Finally, one could also estimate the parameters of the model developed here instead of calibrating it to the moments. I also intend to pursue this in the future.

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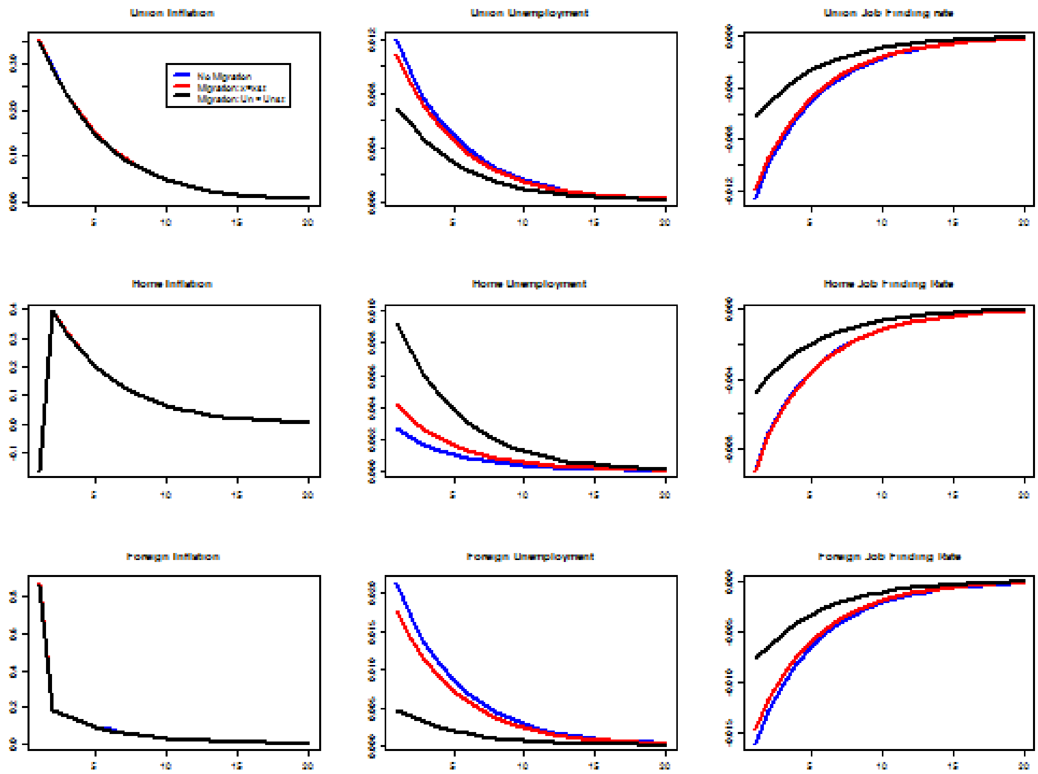
Tables

Table 3: Calibration

	Parameter	Value
Openness	α	0.2
Discount factor	β	0.96
Cost of Price adjustment (Home)	ψ	0.025
Cost of Price adjustment (Foreign)	ψ^*	0.025
Bargaining power (Home)	μ	1
Bargaining power (Foreign)	μ^*	1
Separation rate (Home)	δ^*	0.013
Separation rate (Foreign)	δ^*	0.056
Coef. of autocor. of output (Home)	ρ_F	0.9
Coef. of autocor. output (Foreign)	ρ_I	0.9
Goods elasticity (Home)	ϵ	6
Goods elasticity (Foreign)	ϵ^*	6
Labor Cost (Home)	Γ	7.93 (3.70)
Labor Cost (Foreign)	Γ^*	0.46 (0.67)
Labor cost elasticity wrt to job-finding rate (Home)	γ	1
Labor cost elasticity wrt to job-finding rate (Foreign)	γ^*	1
Labor effort dis-utility (Home)	χ	2.16 (3.11)
Labor effort dis-utility (Foreign)	χ^*	18.03 (10.15)
Elasticity of labor effort wrt unemployment (Home)	κ	0.8
Elasticity of labor effort wrt unemployment (Foreign)	κ^*	0.8

Figures

Figure 1: Negative Productivity Shock in the Foreign Country



Appendix

Equilibrium Conditions of the Model Economy

The economy of the union is characterized by the following equations.

- Variables: $Y_{H,t}; Y_{F,t}; C_t; C_t^*; N_t; N_t^*; x_t; x_t^*; \widetilde{W}_t, \widetilde{W_t^*}; W_t^r; W_r^{*r}; U_t; U_t^*, G_t; G_t^*, MC_t^r; MC_t^{*r}$
- Goods Market Clearing:

$$\begin{aligned} Y_t &= C_t S_t^{\alpha} + G_t H_t + \frac{\psi}{2} \left(\Pi_{H,t} - 1\right)^2 Y_t \\ Y_t^* &= C_t^* S_t^{-\alpha} + G_t^* H_t^* + F_t^* + \frac{\psi^*}{2} \left(\Pi_{F,t} - 1\right)^2 Y_t^* \\ &= C_t S_t^{\alpha-1} + G_t^* H_t^* + F_t^* + \frac{\psi^*}{2} \left(\Pi_{F,t} - 1\right)^2 Y_t^* \\ S_t &= \frac{Y_t - G_t H_t - \frac{\psi}{2} \left(\Pi_{H,t} - 1\right)^2 Y_t}{Y_t^* - G_t^* H_t^* - F_t^* - \frac{\psi^*}{2} \left(\Pi_{F,t} - 1\right)^2 Y_t^*} \end{aligned}$$

- Production Technology:

$$\begin{aligned} Y_{H,t} &= A_t N_t \\ Y_{F,t} &= A_t^* N_t^* \end{aligned}$$

- Aggregate hiring and employment:

$$\begin{aligned} H_t &= N_t - (1-\delta) N_{t-1} \\ H_t^* &= N_t^* - (1-\delta^*) N_{t-1}^* \end{aligned}$$

- Hiring cost:

$$\begin{aligned} G_t &= \Gamma A_t x_t^{\gamma} \\ G_t^* &= \Gamma^* A_t^* x_t^{*\gamma} \end{aligned}$$

- Jobless and looking for employment (in Home and Foreign):

$$\begin{aligned} U_t &= (1 - (1-\delta) N_{t-1}) + (1-\upsilon) \left(1 - (1-\delta^*) N_{t-1}^*\right) \\ U_t^* &= \upsilon \left(1 - (1-\delta^*) N_{t-1}^*\right) \end{aligned}$$

- Job finding rate:

$$\begin{aligned} x_t &= \frac{H_t}{U_t} = \frac{N_t - (1-\delta) N_{t-1}}{(1 - (1-\delta) N_{t-1}) + (1-\upsilon) \left(1 - (1-\delta^*) N_{t-1}^*\right)} \\ x_t^* &= \frac{H_t^*}{U_t^*} = \frac{N_t^* - (1-\delta^*) N_{t-1}^*}{\upsilon \left(1 - (1-\delta^*) N_{t-1}^*\right)} \end{aligned}$$

- Wage Setting (Nash bargained wage):

$$\widetilde{W}_t = \mu A_t \Gamma S_t^{-\alpha} x_t^{\gamma} + \chi C_t L_t^{\phi} - \beta\left(1-\delta\right) E_t \left\{ \frac{C_t}{C_{t+1}} \left[\left(1 - x_{t+1}\right) \left(\mu A_{t+1} \Gamma S_{t+1}^{-\alpha} x_{t+1}^{\gamma} \right) \right] \right\}$$

$$\widetilde{W}_t^{*} = \mu^{*} A_t^{*} \Gamma^{*} S_t^{\alpha} x_t^{*\gamma} + \chi^{*} C_t^{*} L_t^{*\phi} - \delta^{*}(1-\upsilon) f_{e,t}^{*}$$
$$- \left(1 - \delta^{*}\upsilon\right) \beta E_t \left\{ \frac{C_t^{*}}{C_{t+1}^{*}} \left[\left(1 - x_{t+1}^{*}\right) \left(\mu^{*} A_{t+1}^{*} \Gamma^{*} S_{t+1}^{\alpha} x_{t+1}^{*\gamma} \right) \right] \right\}$$

- Real Wage Rigidity:

$$W_t^{r} = \left(\widetilde{W}_t\right)^{1-\nu} \left(W_{ss}\right)^{\nu}$$
$$W_t^{*r} = \left(\widetilde{W}_t^{*}\right)^{1-\nu^{*}} \left(W_{ss}^{*}\right)^{\nu^{*}}$$

- Price Setting:

$$(1-\epsilon) + \epsilon MC_{H,t}^{r} - \psi\left(\Pi_{H,t} - 1\right)\Pi_{H,t} + \psi Q_{t,t+1}\left(\Pi_{H,t+1} - 1\right)\left(\frac{Y_{H,t+1}}{Y_{H,t}}\right)\Pi_{H,t+1}^{2} = 0$$
$$(1-\epsilon^{*}) + \epsilon^{*} MC_{F,t}^{r} - \psi^{*}\left(\Pi_{F,t} - 1\right)\Pi_{F,t} + \psi^{*} Q_{t,t+1}\left(\Pi_{F,t+1} - 1\right)\left(\frac{Y_{F,t+1}}{Y_{F,t}}\right)\Pi_{F,t+1}^{2} = 0$$

- Marginal Costs:

$$MC_t^{r} = \frac{W_t^{r}}{A_t} S_t^{\alpha} + \Gamma x_t^{\gamma} - \beta\left(1-\delta\right) E_t \left\{ \left(\frac{C_t}{C_{t+1}}\right) \left(\frac{S_t}{S_{t+1}}\right)^{\alpha} \left(\frac{A_{t+1}}{A_t}\right) \Gamma x_{t+1}^{\gamma} \right\}$$
$$MC_t^{*r} = \frac{W_t^{*r}}{A_t^{*}} S_t^{-\alpha} + \Gamma^{*} x_t^{*\gamma} - \beta\left(1-\delta^{*}\right) E_t \left\{ \left(\frac{C_t^{*}}{C_{t+1}^{*}}\right) \left(\frac{S_t}{S_{t+1}}\right)^{-\alpha} \left(\frac{A_{t+1}^{*}}{A_t^{*}}\right) \Gamma^{*} x_{t+1}^{*\gamma} \right\}$$

The Model Steady State

The economy of the union is characterized by the following equations.

- Zero inflation steady state: $\Pi_{ss} = \Pi_{H,ss} = \Pi_{F,ss} = 1$
- Variables: $Y_{H,ss}; Y_{F,ss}; C_{ss}; C^*_{ss}; N_{ss}; N^*_{ss}; x_{ss}; x^*_{ss}; W_{ss}, W^*_{ss}; W^r_{ss}; W^{*r}_{ss}; U_{ss}; U^*_{ss}, G_{ss}$ $G^*_{ss}, MC^r_{ss}; MC^{*r}_{ss}$
- Households' conditions:

$$\chi C_{ss} \left(L_{ss}\right)^{\phi} = W^r_{ss} \tag{37}$$

$$\chi^* C^*_{ss} \left(L^*_{ss}\right)^{\phi} = W^{*r}_{ss} - \delta^*(1-\upsilon) \tag{38}$$

$$\beta = \frac{1}{R_{ss}} = Q_{ss} \tag{39}$$

- Goods Market Clearing:

$$\begin{aligned} Y_{ss} &= C_{ss} S^{\alpha}_{ss} + G_{ss} H_{ss} \\ Y^*_{ss} &= C^*_{ss} S^{-\alpha}_{ss} + G^*_{ss} H^*_{ss} + F^*_{ss} \\ &= C_{ss} S^{\alpha-1}_{ss} + G^*_{ss} H^*_{ss} + F^*_{ss} \\ S_{ss} &= \frac{Y_{ss} - G_{ss} H_{ss}}{Y^*_{ss} - G^*_{ss} H^*_{ss} - F^*_{ss}} \end{aligned}$$

- Production Technology:

$$\begin{aligned} Y_{H,ss} &= N_{ss} = C_{ss} S^{\alpha}_{ss} + G_{ss} H_{ss} \\ Y_{F,ss} &= N^*_{ss} = C^*_{ss} S^{-\alpha}_{ss} + G^*_{ss} H^*_{ss} + F^*_{ss} \end{aligned}$$

- Aggregate hiring and employment:

$$\begin{aligned} H_{ss} &= N_{ss} - (1-\delta) N_{ss} = \delta N_{ss} \\ H^*_{ss} &= N^*_{ss} - (1-\delta^*) N^*_{ss} = \delta^* N^*_{ss} \end{aligned}$$

- Hiring cost:

$$\begin{aligned} G_{ss} &= \Gamma x^{\gamma}_{ss} \\ G^*_t &= \Gamma^* x^{*\gamma}_{ss} \end{aligned}$$

- Job finding rate:

$$\begin{aligned} x_{ss} &= \frac{\delta N_{ss}}{(1 - (1-\delta) N_{ss}) + (1-\upsilon)\left(1 - (1-\delta^*) N^*_{ss}\right)} \\ x^*_{ss} &= \frac{\delta^* N^*_{ss}}{\upsilon \left(1 - (1-\delta^*) N^*_{ss}\right)} \end{aligned}$$

- Jobless and looking for employment (in Home and Foreign):

$$\begin{aligned} U_{ss} &= (1-(1-\delta)N_{ss}) + (1-\upsilon)\,(1-(1-\delta^*)N^*_{ss}) \\ U^*_{ss} &= \upsilon\,(1-(1-\delta^*)N^*_{ss}) \end{aligned}$$

- Wage Setting (Nash bargained wage):

$$\begin{aligned} W_{ss} &= \mu\Gamma S_{ss}^{-\alpha} x_{ss}^{\gamma} + \chi C_{ss} N_{ss}^{\phi} - \beta\,(1-\delta)\,(1-x_{ss})\left(\mu\Gamma S_{ss}^{-\alpha} x_{ss}^{\gamma}\right) \\ W^*_{ss} &= \mu^*\Gamma^* S_{ss}^{\alpha} x_{ss}^{*\gamma} + \chi^* C^*_{ss} N_{ss}^{*\phi} - \delta^*\upsilon F^*_{ss} - (1-\delta^*(1-\upsilon))\,\beta(1-x^*_{ss})\mu^*\Gamma^* S_{ss}^{\alpha} x_{ss}^{*\gamma} \end{aligned}$$

- Real Wage Rigidity:

$$\begin{aligned} W^r_{ss} &= W_{ss} \\ W^r_{ss} S^{\alpha}_{ss} &= \mu\Gamma x_{ss}^{\gamma} + \chi C_{ss} N_{ss}^{\phi} S^{\alpha}_{ss} - \beta\,(1-\delta)\,(1-x_{ss})\,(\mu\Gamma x_{ss}^{\gamma}) \\ W^{*r}_{ss} &= W^*_{ss} \\ W^{*r}_{ss} S^{-\alpha}_{ss} &= \mu^*\Gamma^* x_{ss}^{*\gamma} + \chi^* C^*_{ss} N_{ss}^{*\phi} S^{-\alpha}_{ss} - \delta^*(1-\upsilon)F^*_{ss} - (1-\delta^*\upsilon)\,\beta(1-x^*_{ss})\mu^*\Gamma^* x_{ss}^{*\gamma} \end{aligned}$$

- Price Setting:

$$\begin{aligned} MC^r_{ss} &= \frac{\epsilon-1}{\epsilon} = \varrho \\ MC^r_{F,ss} &= \frac{\epsilon^*-1}{\epsilon^*} = \varrho^* \end{aligned}$$

- Marginal Costs:

$$\begin{aligned} MC^r_{ss} &= W^r_{ss} S^{\alpha}_{ss} + \Gamma x_{ss}^{\gamma} - \beta\,(1-\delta)\,\Gamma x_{ss}^{\gamma} \\ &= \mu\Gamma x_{ss}^{\gamma} + \chi C_{ss} N_{ss}^{\phi} S^{\alpha}_{ss} - \beta\,(1-\delta)\,(1-x_{ss})\,(\mu\Gamma x_{ss}^{\gamma}) + \Gamma x_{ss}^{\gamma} - \beta\,(1-\delta)\,\Gamma x_{ss}^{\gamma} \\ \frac{1}{\rho} &= (1+\mu)\,\Gamma x_{ss}^{\gamma} + \chi C_{ss} N_{ss}^{\phi} S^{\alpha}_{ss} - \beta\,(1-\delta)\,\Gamma x_{ss}^{\gamma}\,[1+\mu(1-x_{ss})] \end{aligned}$$

$$\begin{aligned} W^{*r}_{ss} &= W^*_{ss} \\ MC^{*r}_{ss} &= W^r_{ss} S^{-\alpha}_{ss} + \Gamma^* x_{ss}^{*\gamma} - \beta\,(1-\delta^*)\,\Gamma^* x_{ss}^{*\gamma} \\ &= \mu^*\Gamma^* x_{ss}^{*\gamma} + \chi^* C^*_{ss} N_{ss}^{*\phi} S^{-\alpha}_{ss} - \delta^*(1-\upsilon)F^*_{ss} - (1-\delta^*\upsilon)\,\beta(1-x^*_{ss})\mu^*\Gamma^* x_{ss}^{*\gamma} + ... \\ &\quad ...\Gamma^* x_{ss}^{*\gamma} - \beta\,(1-\delta^*)\,\Gamma^* x_{ss}^{*\gamma} \\ \frac{1}{\rho^*} &= (1+\mu^*)\Gamma^* x_{ss}^{*\gamma} + \chi^* C^*_{ss} N_{ss}^{*\phi} S^{-\alpha}_{ss} - \delta^*(1-\upsilon)F^*_{ss}... \\ &\quad ... - (1-\delta^*\upsilon)\,\beta(1-x^*_{ss})\mu^*\Gamma^* x_{ss}^{*\gamma} - \beta\,(1-\delta^*)\,\Gamma^* x_{ss}^{*\gamma} \end{aligned}$$